Lecture 2 exercises: Matrices

1. Multiply the following matrices:

   $(a.)$ $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} -5 & 1 \\ 0 & 2 \end{bmatrix}$ $(b.)$ $\begin{bmatrix} 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix}$ $(c.)$ $\begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \end{bmatrix}$

2. Multiply:

   $$\begin{bmatrix} 2 & 3 & 0 \\ -1 & 4 & 1 \end{bmatrix} \begin{bmatrix} -5 & 1 & 4 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

3. For two matrices A and B and a scalar c, is it always true that $(cA)B = c(AB)$ (assuming that A and B can be multiplied)?

4. If we multiply two matrices, one of which is the null matrix, is the answer always the null matrix? If we multiply two matrices and the answer is the null matrix, is it then true that one of the input matrices is the null matrix?

5. Find two 2×2 matrices that do not have any zero entries, but such that their product is the zero matrix.

6. Solve the following set of equations using Gaussian elimination:

   $$\begin{aligned} 6x + 4y &= 9 \\ 3x + 5y &= -2 \end{aligned}$$

7. Solve the following set of equations using Gaussian elimination:

   $$\begin{aligned} x + y - 2z &= 2 \\ 2x - y + z &= 3 \\ -x + 2y + 4z &= 5 \end{aligned}$$

8. Does the following set of linear equations have a solution?

   $$\begin{aligned} 7x + 4y &= 12 \\ 3x + 5y &= -2 \\ x - y &= 4 \end{aligned}$$

9. What is the solution of the following set of equations?

   $$\begin{aligned} 5x + 11y - z &= -8 \\ 3x + 5y + z &= -2 \\ x - y + 3z &= 4 \end{aligned}$$

   What if the last equation equals 3 instead of 4, what is then the solution?

10. Compute the inverse by Gaussian elimination: $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

    How can you verify easily that your answer is correct? Perform this verification.

11. Compute the inverse by Gaussian elimination: $\begin{bmatrix} 2 & 3 & -1 \\ -1 & 4 & 0 \\ 7 & 2 & -3 \end{bmatrix}$

12. Is it possible to invert a 1×1 matrix?